Journal of Computer Technology and Applied Mathematics
Journal Home: http://jctam.suaspress.org | CODEN: JCTAAD
Vol. 1, No. 4, 2024 | ISSN 3007-4126 (Print) | ISSN 3007-4134 (Online) | ISSN 3007-4142 (Digital)

Game Theory: Concepts, Applications, and Insights from Operations Research

CHE, Chang [1*] TIAN, Junchi [1]

[1] The George Washington University, USA

**** CHE, Chang is the corresponding author, E-mail: cche57@gwmail.gwu.edu***

Abstract: Game theory is a branch of complex system analysis that deals with the mathematical modeling of strategic interactions among rational agents. It has been widely used across various fields, such as economics, social sciences, computer science, and operational research. This paper provides an overview of game theory, discussing its foundational concepts, including non-cooperative and cooperative games, Nash equilibrium, and mixed strategies. The study explores extensive-form, repeated, Bayesian, and coalitional games, highlighting applications in operational research and the implications for decision-making under competition.

Keywords: Game Theory, Nash Equilibrium, Mixed Strategy, Operational Research, Cooperative Games.

Disciplines: Applied Mathematics. **Subjects:** Game Theory.

DOI: https://doi.org/10.5281/zenodo.13924241 **ARK:** https://n2t.net/ark:/40704/JCTAM.v1n4a07

1 1 INTRODUCTION

1.1 BACKGROUND

Game theory is a topic under complex system analysis and studies the mathematical modeling for decision-making. Its application can cover various fields including social science, computer science as well as operational research. The basic of game theory is describing that different players are going to play a game with some selective actions under a specific designed scheme, where the players are expecting to receive payoffs based on their actions. The main concept is divided into noncooperative game theory and cooperative game theory. The former one means different players are performing individually and selfishly. Everyone's behaviors should be directly modeled. While the latter one, cooperative game theory, refers to more abstract and axiomatic analysis of bargains or behaviors that players might reach, without explicitly modeling the processes [4]. Based on the fact that non-cooperative games are more general in practices, the cooperative games could be analyzed through the approaches of solving non-cooperative games with the scenario-based assumptions. Other kinds of divisions are symmetric or asymmetric games, zero-sum or non-zero-sum games, simultaneous or sequential games, etc.

Game theory has a long history. Discussions about games and underpinning mathematics had begun much earlier before the modern game theory. In 1564, Cardano wrote the book on Games of Chance and published it in 1663. Then, Pascal and Huygens studied on reasoning of the Games of Chance's framework in 1650s. In the 18th and 19th century, some researchers and scientists such as Charles Waldegrave, Antoine Augustin Cournot further developed this area, and Cournot started to present a solution that is the Nash equilibrium of the game. In 1913, Ernst Zermelo proved that the strict determination of optimal chess strategy, therefore, paved the way for more general theorems. Later on, the Danish economist Frederik Zeuthen proved the winning strategy of mathematical model by Brouwer's fixed point theorem. Also, Borel conjectured that the mixed strategy equilibria is not existent in finite two-person zero-sum games, which was verified false by von Neumann [12].

In 1928, John von Neumann published the paper On the Theory of Games of Strategy, which led Game Theory to become a unique research field. Several years later, in 1944, he published the book Theory of Games and Economic Behavior. This work foundationally explains how to explore the mutually consistent solutions for two-person zero-sum games. The well-known prisoner's dilemma was discussed in 1950, and its corresponding experiments are conducted by Merrill M. Flood and Melvin Dresher. Almost at the same time, John Nash defined Nash equilibrium that is a criterion to be applied to various games. In 1950s, game theory has experienced a fast growing. Many different types of games people are using nowadays are defined at that time [18].

1.2 GAMES IN NORMAL FORM

Here, the normal form of a game will be defined [10].

Considering the following settings:

- The set of players is $N = \{1,2,3, \dots, n\}$;
- The set of actions for each player refer to the strategies. The profile of these strategies or actions is defined as $a = \{a_1, a_2, a_3, \dots, a_n\}$;
- The players' payoff can be defined as a function of action a_i: $u_i(a_i): A \rightarrow R$.

Based on the above settings the famous example Prisoners' Dilemma Game is shown:

TABLE 1. PRISONERS' DILEMMA

Prisoners' Dilemma	Player 2		
Player 1	Actions (A)	C	D
	C	-1, -1	-3, 0
	D	0, -3	-2, -2

Where the situation is that two offenders are interrogated separately, Player 1 and Player 2 do not know the actions of each other. There are two actions for players(offenders) to select: C means "cooperate", while D means "defect". From the table, it can be found that if both of the players act as cooperate, then both of them will have the 1-year prison sentence. However, if anyone of them acts as defect, the player who defects will not have prison sentence, but the opposite play who still acts cooperate will face a 3-years sentence. If both of them defect with each other, both of them will be given 3-years prison sentence. In this example, the payoffs of players' actions are their received prison sentence. The length can be scaled properly and this will not influence the model [15].

1.3 NASH EQUILIBRIUM

Given a game and its formulated model, the best choices can be predicted to maximize the payoff of players. The dominant strategies are powerful to analyze this problem. The dominant strategy for a player is defined as the strategy that generates the highest or greatest payoff of any available strategies from actions of other players. The strictly dominant strategy stands for the dominant strategy is strictly larger than any others (the only one maximum). In this way, the analysis of games can be quite easy with help of dominant strategy. Besides, the Pareto optimality and Pareto dominates are also used to describe the best response or maximum payoff. The definition of Pareto optimality is that an outcome is a Pareto optimality if no other outcome Pareto dominates it, where the Pareto dominate stands for a choice is reasonably better than the other one [4]. Pareto optimality is not as strong as dominant strategy. A game can have more than one Pareto optimality, and even can have none Pareto optimality. However, when dominant strategies do not exist but we still want to explore the best response. The Nash equilibrium is presented.

Nash equilibrium describes a profile of strategy equips with the best responds for each player against the other players' equilibrium strategy. In other words, it can be written as results in the highest available payoff with action a_i, where a profile of strategy $a \in A$: $u_i(a_i, a_{-i}) \geq u_i(a_i', a_{-i})$. A Nash equilibrium needs the actions taken by each player to be the best against the equilibrium actions of other players, rather than all the possible actions. This is to say that a set of dominant strategies is a subset of Nash equilibrium. In the prisoner's dilemma, the action set (D,D) for player 1 and player 2 is, therefore, a Nash equilibrium (Holt, 2004).

1.4 MIXED STRATEGY

The abovementioned strategies are pure strategies that indicates only a single action is chosen by players. In contrast, not all games exist pure strategy equilibrium. Then, mixed strategies should be considered. An example is shown as follows:

TABLE 2. A PENALTY-KICK GAME

A Penalty-Kick Game	Goalie		
Kicker	Actions (A)	L	R
	L	-1, 1	1, -1
	R	1, -1	-1, 1

This example describes a football game, where the kicker is going to shoot the ball towards left(L) or right(R), and the goalie is also going to defend towards these two directions. If these two players act in the same direction, the goalie wins but the kicker loses. But if they choose the opposite directions, the kicker wins while the goalie loses [2].

After observation and analysis, there is no Nash equilibrium for the single action of the players. In order to solve this problem, the kicker and goalie are assumed to act towards one direction randomly and respectively. This formulates a randomized strategy, or so-called mixed strategy. Comparing with the pure strategy, mixed strategy extends the strategy spaces to infinite with the proper defined probabilities of different actions. For a finite set of strategies of a certain game, pure strategy may not exist, but mixed strategy must exist.

Let a player i has mixed strategy with a distribution s_i on a_i, the action a_i's probability is $s_i(a_i)$. Then, the set of mixed strategy for Nash equilibrium can be expressed as:

$$\sum_{a} \left(\prod_{j} s_j(a_j) \right) u_i(a) \geq \sum_{a_{-i}} \left(\prod_{j \neq i} s_j(a_j) \right) u_i(a_i', a_{-i})$$

This formula describes that the no other player can create higher payoff with a distinct strategy than mixed strategy for Nash equilibrium given a certain strategy of other players. To be mentioned, since the mixed strategy has probabilities of actions, pure strategy is a special case of mixed strategy with the probability is equal to 1.

In terms of computing the mixed strategy, one of the most important concepts is that making any player to be indifferent with their actions, when this player is facing others' strategy's probabilities. In other words, with actions'

Journal of Computer Technology and Applied Mathematics
Journal Home: http://jctam.suaspress.org | CODEN: JCTAAD
Vol. 1, No. 4, 2024 | ISSN 3007-4126 (Print) | ISSN 3007-4134 (Online) | ISSN 3007-4142 (Digital)

probabilities of other players, one specific player must have equal summed payoff from its different actions. For example, if one player 1 has two actions a_1 and a_2. Then, based on the abovementioned notions, the equivalent formula is $u_1(a_1) = u_1(a_2)$. This conclusion is deduced from mixed strategy's randomization, which is necessary to consider that player is indifferent or no preference on which action to choose. As a result, the equal summed payoff of different actions is the support of player's indifferent.

2 SUBJECTS OF GAMES

2.1 EXTENSIVE-FORM GAMES

If considering the timing of decision-making, the game becomes extensive-form games, which express the sequence of actions explicitly for different players. In extensive-form games, the actions selected by the players are not at a single time. Some of the players will act in the game followed by the other players select actions based on the previous ones. Extensive-form games formulate a decision-tree or a game tree and can be regarded as a dynamic game. It includes the following information: (1) the set of participated players; (2) the set of players' available actions; (3) the timing history or sequence of the game; (4) the information set players are grasping. Based on the information set, the game can be further developed to complete information and incomplete information; (5) the probability distribution of history games and actions; (6) the effective function of participated players. Comparing with the strategy games in the previous sections, each extensive-form game can correspond to a strategy game, but the reverse is not consistent [11].

An illustration of the extensive-form game is shown as follow:

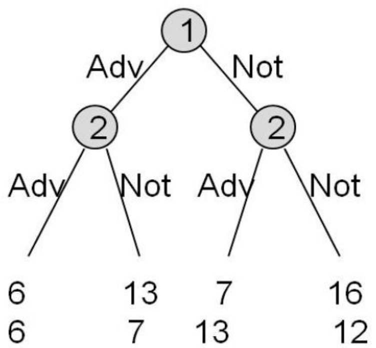

FIGURE 1. ADVERTISING CHOICES OF TWO COMPETITORS

This game is originally from strategy game. It describes two firms are planning to advertise or not for a business. Both of the firm 1 and firm 2 have two actions: advertising or not. In the figure, the tree starts from the first player in circled notion. After the player 1(firm 1) make its decision, which is presented by the edge of the tree, the player 2(firm 2) will take actions based on game history. The group of actions every player can take here is same. Therefore, a game tree has formed through the nodes of players and branches of actions. After all the players and actions are finished their processes, the branch goes to termination. Each termination of the branch shows the payoff of the actions and players in the game. For instance, if both of the firms select not to advertise, then the game tree arrives at the terminated branch of (16,12), which indicates the payoff for the first player, firm 1, is 16 and payoff for firm 2 is 12. By this way, all the possibilities are illustrated clearly in the figure.

In this case, firm 2 will learn from firm 1's action and make decision. Applying the Nash equilibrium in pure strategy, there are two statuses hold. One is firm 1 advertise, firm 2 not. The other one is firm 1 does not advertise, while firm 2 advertise no matter what action firm 1 does. Actually, the latter Nash equilibrium is not credible since it emphasizes that firm 2 will not consider firm 1's action. However, in extensive-form game, if firm 2 has already seen firm 1 advertises, firm 2 will not advertise given that this decision will lead to the lowest payoff of both firms.

To work out a credible solution, backward induction is utilized for extensive-form games. The decision-making of backward induction starts from the terminations of these branches. If firm 1 does not advertise, firm 2 will consider to advertise since in this way the payoff is 13 that is higher than 12. If firm 1 advertises, then, firm 2 will select to not advertise to pursue the payoff of 7 that is higher than 6. Based on these known conditions. Firm 1 will compare two branches:

$$\{Firm\ 1\ Adv, Firm\ 2\ Not\} ==> Payoff: \{13,7\},$$

and

$$\{Firm\ 1\ Not, Firm\ 2\ Adv\} ==> Payoff: \{7,13\}.$$

The former one provides payoff of 13 for firm 1, while the latter one offers payoff of 7. Therefore, the firm 1 tends to select to advertise and firm 2 will not advertise. The final payoff will be {13,7} respectively. This result leads that not advertising is no longer the dominant strategy for firm 1, because the firm 2's action could be influenced by what firm 1 has done.

Generally, the solution concept of this game is known as subgame perfect equilibrium [16]. A subgame is a subsection of the extensive-form game from a specific node. The terminology subgame perfection means that all the subgames compose Nash equilibrium. It is also worth noting that the first player does not have common advantages or disadvantages. The sequence of players' decision-making should follow the mechanism of games.

2.2 REPEATED GAMES

Repeated game is a special case of extensive-form game consists of a number of repetitions. Every single repeated game is called base game or stage game. Some examples of repeated games include the athletic contests, like tennis, badminton, etc. Repeated game is the superposition of many strategy games or extensive-form games with many rounds, where each round is a solely stage game and is equivalent to the others [1].

Journal of Computer Technology and Applied Mathematics
Journal Home: http://jctam.suaspress.org | CODEN: JCTAAD
Vol. 1, No. 4, 2024 | ISSN 3007-4126 (Print) | ISSN 3007-4134 (Online) | ISSN 3007-4142 (Digital)

The based game can be represented by the notion G, the number of repetitions can be written as notion T. In this way, the repeated games can be defined as $G(T)$, which means the repeated game with T repetitions. There are three basic characteristics of repeated games: (1) Every stage game does not have the physical link, and is independent with each other. The previous games will not change the latter games. (2) All the participated players can observe the history of games. (3) The payoff of the participated player is the weighted average sum or present discount value of all the repeated stage games. Since the player's current decision will be influenced based on their observation towards the history games, the Nash equilibrium is a greatly complicated problem to analyze. Both of the psychological factors and game itself impose impacts on players' decision-making space.

According to the T repetitions, the repeated games can be further divided into finitely repeated games and infinitely repeated games. The former one is straight forward to understand, since T is a finite value. The concept of subgame perfect Nash equilibrium and the technique of backward induction are suitable to finitely repeated games. The payoff is the sum of the relevant discrete stage games. Based on the formula of interest computing: $\delta = \frac{1}{1+\gamma}$, in which γ is decided by the periodic market. The total payoff of a T finitely repeat game is:

TABLE 3. HAWK-DOVE GAME

Hawk-Hawk	Hunting	Escaping
Hunting	-2,-2	2,-1
Escaping	-1,2	-1,-1
Dove-Dove	Hunting	Escaping
Hunting	1,1	2,0
Escaping	0,2	0,0
Hawk-Dove	Hunting	Escaping
Hunting	2,-2	2,0
Escaping	-1,2	-1,0

$$G(\delta, T) = \pi_1 + \delta\pi_2 + \delta^2\pi_3 + \cdots + \delta^{T-1}\pi_T$$
$$= \sum_{t=1}^{T} (\delta^{T-1}\pi_T)$$

However, when considering the infinitely repeated games or finitely repeated games with uncertain repetitions T, the situation becomes hard to handle given that the branches of decision tree for this kind of games do not have a clearly defined termination. As a result, the backward induction and subgame perfect Nash equilibrium are no longer applicable. Here, the reputation of agents involved in the games and some techniques of punishments are introduced to modeled the utility function. Take the prisoner's dilemma games with repeated iterations as an example. Every player will act selfishly to maximize their own profits. The selfish action may deviate from the cooperative strategy that could potentially make both of the players get the best results. However, if there is a trigger strategy that make the payoff of selfish actions from players continuously decay with the running of repeated game. After observing the history of games, the players will realize this punishment and finally tend to the cooperative strategy. This leads to maintaining socially optimal equilibrium in repeated games, and "Folk Theorems" for establishing players' preference in infinitely repeated games [18]. Two essential forms of modeling of players' utility functions:

Limit of means: $U_i = inf \frac{1}{T} \sum_{t=0}^{T} u_i(x_t)$.

Discounting: $U_i = \sum_{t\geq 0} \delta^t u_i(x_t)$.

2.3 BAYESIAN GAMES

Bayesian game is a kind of incomplete information, where the players do not know the exact payoff functions, but have a belief of these payoffs. Their beliefs are expressed as the form of probability distributions. The probability distribution of players' beliefs obeys the Bayes' rule. The incomplete information of players stands for one player does not know other players' types and specific attributes. To address this problem, John C. Harsanyi [18] introduced an extra player, nature, to help distribute different types of players. This theorem is based on the probability distribution and assign the probability to players. In this way, the original incomplete information game has been transferred to imperfect information game. The best responds and indifferent conditions can be, therefore, adopted [5].

The following is an example of Bayesian games namely Hawk-dove game, in Table 3.

Here, the players are Hawk and Dove, two kinds of bird, with the above illustrated payoff matrix. Each kind of bird will act hunting or escaping towards the other players (other bird). Now, there is a bird starts this Bayesian game. It does not know what bird it will encounter. Based on the Harsanyi's transformation, the proportion of Hawk and Dove in the whole group of bird can be 50%:50% following the probability distribution. This is what exactly the nature work in Bayesian games. The proportions can vary under various scenarios. Then, the Bayesian game for Hawk and Dove can be concluded respectively:

TABLE 4. BAYESIAN GAME FOR HAWK

Probability	Opposite	50%		50%		
Self	Hawk-Bird	H-hunting (a)	H-escaping (1-a)	D-hunting (b)	D-escaping (1-b)	Expectations
a	hunting	-2,-2	2,-1	2,-2	2,0	2-2a
1-a	escaping	-1,2	-1,-1	-1,2	-1,0	-1

The expectation of payoff for a Hawk chooses hunting is:

$E(hunting) = 0.5[-2a + 2(1-a)] + 0.5[2b +$

Journal of Computer Technology and Applied Mathematics
Journal Home: http://jctam.suaspress.org | CODEN: JCTAAD
Vol. 1, No. 4, 2024 | ISSN 3007-4126 (Print) | ISSN 3007-4134 (Online) | ISSN 3007-4142 (Digital)

$2(1-b)] = 2-2a$.

$E(escaping) = 0.5[-a+a-1] + 0.5[-b+b-1] = -1$.

Similarly,

TABLE 5. BAYESIAN GAME FOR DOVE

Probability	Opposite	50%		50%		
Self	Dove-Bird	H-hunting (a)	H-escaping (1-a)	D-hunting (b)	D-escaping (1-b)	Expectations
b	hunting	-2,2	2,-1	1,1	2,0	2-2a-0.5b
1-b	escaping	0,2	0,-1	0,2	0,0	0

In summary, the expectation functions for Hawk and Dove are:

$E(Hawk) = a(2-2a) - (1-a) = -2a^2 + 3a - 1$;

$E(Dove) = b(2-2a-0.5b) - 0 = -0.5b^2 + 2b - 2ab$;

The $E(Hawk)$ has the maximum at a=3/4 with the value of 1/8, then, $E(Dove)$ has the maximum at b=1/2 with the value of 1/8. Given that both of the players have realized their own maximized payoff with the above mixed strategy, this status is called Bayesian Nash equilibrium. Such an equilibrium is evidently related to the payoff matrix when the players encounter with each other. The Bayesian Nash equilibrium is formally defined as the maximized payoff's strategies of players under the beliefs about the nature. Moreover, the proportion of 1:1 in this case is called the evolutionarily stable strategy for the species.

2.4 Coalitional Games

The coalitional game (also named as cooperative game) is a concept emphasizes cooperative among individual players and gathers players together to compete with other group of players. The cooperation is formed with the help of some law contracts or self-enforcing. The coalitional game is opposite to the traditional non-cooperative game theory that analyzes Nash equilibrium. Since the non-cooperative games are more general in the reality, the cooperative games can be considered by the approaches of non-cooperative ones. More importantly, a coalitional game concentrate on how its group can cooperate well.

Coalitions aims at getting greater payoffs for each player in the group. Transferable utility of coalitions transfers N players' payoff into the group's payoff $v(S)$, where $S \in N$. The grand coalition refers to the set of all players in N, and has the highest payoff. One of the key properties is Superadditive game: $v(S \cup T) \geq v(S) + v(T)$, for $S, T \in N$; $S \cap T = \emptyset$. Based on these definitions, analyzing coalition games become analyzing the methods of divide its payoff.

One method called the Shapley Value identifies the fairness of payoff division. Under this method, the received payments are closely related to each member's marginal contributions. Three axioms are highlighted for any v. (1) Symmetry: if i and j are interchangeable then $\varphi_i(N,v) = \varphi_j(N,v)$; (2) Dummy: if i is a dummy player then $\varphi_i(N,v) = 0$; (3) Additivity: $\varphi_i(N, v_1+v_2) = \varphi_i(N,v_1) + \varphi_i(N,v_2)$ for each i, where the game (N, v_1+v_2) is defined by $(v_1+v_2)(S) = v_1(S) + v_2(S)$ for every coalition S [4].

The following expression shows how Shapley Value divides payoffs for members of coalition with the weighted system:

$$\phi_i(N,v) = \frac{1}{N!} \sum_{S \subseteq N \setminus \{i\}} |S|!(|N|-|S|-1)! \left[v(S \cup \{i\}) - v(S) \right].$$

A coalitional game (N,v) has only one unique $x(v) = \varphi(N,v)$ divides the whole payoff of coalition and satisfies all three axioms above.

From the perspective of stability, the method of core is proposed. Given that not all members in the coalition will trust the stability of cooperation, while some of members may prefer the smaller coalitions. To address the problem of concerns of stability, the core, x, in coalitional game(N,v) is defined as $\forall S \in N, \sum_{i \in S} x_i \geq v(S)$. This definition is analogous to Nash equilibrium.

3 DISCUSSIONS AND CONCLUSIONS

Operational research is always an effective tool to solve the problems in game theory. For example, the Nash equilibrium of games can be modeled as an optimization problem by linear programming. Generally, game theory is a subset of operational research, and makes some problems in operational research more specific [14].

In supply chain management, game theory offers a robust analytical framework to model decision-making in areas such as inventory management [3], seller-buyer relationships, and manufacturer-retail networks [6]. During disruptions like the Covid-19 pandemic, game theory has proven instrumental in addressing labor constraints and ensuring supply chain resilience by predicting the behavior of different stakeholders under stress. In transportation, game theory is used to optimize logistics networks [8], manage congestion [19], and analyze competitive dynamics between carriers or logistics service providers [13]. It enables more informed decision-making that accounts for the strategic interactions between multiple stakeholders, leading to enhanced efficiency and cost-effectiveness.

Moreover, game theory also finds application in coordinating transportation systems to manage shared resources effectively, such as in dynamic routing [7], scheduling, and resource allocation problems [17]. By leveraging cooperative and non-cooperative game models, the transportation sector can achieve better integration of supply chain activities, particularly in multi-modal transport, where decisions by individual actors can significantly impact overall network efficiency. These game-theoretic models help stakeholders align their strategies, resulting in improved service quality, cost minimization, and efficient use of transportation infrastructure [20]. By modeling cooperative strategies among stakeholders, game theory can facilitate the adoption of green initiatives, such as carbon emission reduction, energy-efficient logistics, and sustainable sourcing practices [21-29].

In daily life, we often see some behaviors that have the nature of struggle or competition with each other. Behaviors that are competitive or confrontational are called countermeasures. In this type of behavior. The parties participating in the struggle or competition each have different goals and interests. In order to achieve their respective goals and interests, all parties must consider various possible action plans of their opponents and try to choose the most beneficial or most reasonable plan for themselves. Game theory is a mathematical theory and method that studies whether there is the most reasonable plan of action for all parties involved in the strategy, and how to find this reasonable plan of action.

ACKNOWLEDGMENTS

The authors thank the editor and anonymous reviewers for their helpful comments and valuable suggestions.

FUNDING

Not applicable.

INSTITUTIONAL REVIEW BOARD STATEMENT

Not applicable.

INFORMED CONSENT STATEMENT

Not applicable.

DATA AVAILABILITY STATEMENT

The original contributions presented in the study are included in the article/supplementary material, further inquiries can be directed to the corresponding author.

CONFLICT OF INTEREST

The authors declare that the research was conducted in the absence of any commercial or financial relationships that could be construed as a potential conflict of interest.

PUBLISHER'S NOTE

All claims expressed in this article are solely those of the authors and do not necessarily represent those of their affiliated organizations, or those of the publisher, the editors and the reviewers. Any product that may be evaluated in this article, or claim that may be made by its manufacturer, is not guaranteed or endorsed by the publisher.

AUTHOR CONTRIBUTIONS

Not applicable.

ABOUT THE AUTHORS

CHE, Chang

The George Washington University, US.

TIAN, Junchi

The George Washington University, US.

REFERENCES

[1] Alesina, A. (1987). Macroeconomic policy in a two-party system as a repeated game. The Quarterly journal of economics, 102(3), 651-678.

[2] Azar, O. H.-E. (2011). Do soccer players play the mixed-strategy Nash equilibrium? Applied Economics, 43(25), 3591-3601.

[3] Yan, Y., Deng, Y., Cui, S., Kuo, Y. H., Chow, A. H., & Ying, C. (2023). A policy gradient approach to solving dynamic assignment problem for on-site service delivery. Transportation Research Part E: Logistics and Transportation Review, 178, 103260.

[4] Coursera. (2021, 10 18). 7-3 The Shapley Value. Retrieved from Game Theory: https://www.coursera.org/learn/game-theory-1/lecture/fWSQi/7-3-the-shapley-value

[5] Dekel, E. F. (2004). Learning to play Bayesian games. Games and Economic Behavior, 46(2), 282-303.

[6] Yan, Y., Chow, A. H., Ho, C. P., Kuo, Y. H., Wu, Q., & Ying, C. (2022). Reinforcement learning for logistics and supply chain management: Methodologies, state of the art, and future opportunities. Transportation Research Part E: Logistics and Transportation Review, 162, 102712.

[7] Liu, T., & Meidani, H. (2024). End-to-end heterogeneous graph neural networks for traffic assignment.

Transportation Research Part C: Emerging Technologies, 165, 104695.

[8] Cheng, X., & Lin, J. (2024). Is electric truck a viable alternative to diesel truck in long-haul operation?. Transportation Research Part D: Transport and Environment, 129, 104119.

[9] Holt, C. A. (2004). The Nash equilibrium: A perspective. Proceedings of the National Academy of Sciences, 101(12), 3999-4002.

[10] Jackson, M. O. (2011, December 5). A Brief Introduction to the Basics of Game Theory. Retrieved from SSRN: https://ssrn.com/abstract=1968579 or http://dx.doi.org/10.2139/ssrn.1968579

[11] Kroer, C. &. (2014, June). Extensive-form game abstraction with bounds. In Proceedings of the fifteenth ACM conference on Economics and computation, pp. 621-638.

[12] Myerson, R. B. (2013). Game theory. Harvard university press.

[13] Cheng, X., Mamalis, T., Bose, S., & Varshney, L. R. (2024). On Carsharing Platforms With Electric Vehicles as Energy Service Providers. IEEE Transactions on Intelligent Transportation Systems.

[14] Ormerod, R. J. (2010). OR as rational choice: A decision and game theory perspective. Journal of the Operational Research Society, 61(12), 1761-1776.

[15] Osborne, M. J. (2004). An introduction to game theory (Vol. 3, No. 3). New York: Oxford university press.

[16] Selten, R. (1975). Reexamination of the Perfectness Concept for Equilibrium Points in Extensive Games. International Journal of Game Theory, pp. 4:25 - 55.

[17] Liu, T., & Meidani, H. (2024). Neural network surrogate models for aerodynamic analysis in truck platoons: Implications on autonomous freight delivery. International Journal of Transportation Science and Technology.

[18] Wikiperdia. (2021, 10 13). Game theory. Retrieved from https://en.wikipedia.org/wiki/Game_theory#Cooperative_/_non-cooperative

[19] Cheng, X., Nie, Y. M., & Lin, J. (2024). An autonomous modular public transit service. Transportation Research Part C: Emerging Technologies, 104746.

[20] Liu, T., & Meidani, H. (2024). Graph Neural Network Surrogate for Seismic Reliability Analysis of Highway Bridge Systems. Journal of Infrastructure Systems, 30(4), 05024004.

[21] Guo, G., Li, X., Zhu, C., Wu, Y., Chen, J., Chen, P., & Cheng, X. (2025). Establishing benchmarks to determine the embodied carbon performance of high-speed rail systems. Renewable and Sustainable Energy Reviews, 207, 114924.

[22] Che, C., Lin, Q., Zhao, X., Huang, J., & Yu, L. (2023, September). Enhancing Multimodal Understanding with CLIP-Based Image-to-Text Transformation. In Proceedings of the 2023 6th International Conference on Big Data Technologies (pp. 414-418).

[23] Che, C., Hu, H., Zhao, X., Li, S., & Lin, Q. (2023). Advancing Cancer Document Classification with R andom Forest. Academic Journal of Science and Technology, 8(1), 278-280.

[24] Huang, Z., Zheng, H., Li, C., & Che, C. (2024). Application of machine learning-based k-means clustering for financial fraud detection. Academic Journal of Science and Technology, 10(1), 33-39.

[25] Huang, Z., Che, C., Zheng, H., & Li, C. (2024). Research on Generative Artificial Intelligence for Virtual Financial Robo-Advisor. Academic Journal of Science and Technology, 10(1), 74-80.

[26] Che, C., Huang, Z., Li, C., Zheng, H., & Tian, X. (2024). Integrating generative AI into financial market prediction for improved decision making. Applied and Computational Engineering, 64, 155-161.

[27] Che, C., Li, C., & Huang, Z. (2024). The Integration of Generative Artificial Intelligence and Computer Vision in Industrial Robotic Arms. International Journal of Computer Science and Information Technology, 2(3), 1-9.

[28] Liu, H., Wang, C., Zhan, X., Zheng, H., & Che, C. (2024). Enhancing 3D Object Detection by Using Neural Network with Self-adaptive Thresholding. arXiv preprint arXiv:2405.07479.